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Determination of Third- and Fourth-Order Longitudinal Elastic Constants by Shock Compression Techniques—Application to Sapphire and Fused Quartz*

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A number of solids sustain large elastic compressions under shock-wave loading. In these solids, measurements of the stress and compression in the direction of shock propagation can be used to calculate both third- and fourth-order longitudinal elastic constants if measurements are carried out over a wide range of compressions. Only limited measurements of fourth-order constants have been previously determined by other techniques. Determinations of third-order constants under these large elastic compressions afford the opportunity to test the applicability of the finite-strain formulation of constitutive relations. A general method for calculating these third- and fourth-order constants is presented and applied to shock compression data for sapphire and fused quartz. For sapphire, it is found that $C_{111} \approx C_{333} = -(3.3 \pm 0.3) \times 10^4$ kbar and $C_{1111} \approx C_{3333} = +(5.0 \pm 1.5) \times 10^5$ kbar. For fused quartz, it is found that $C_{111} = +(5.5 \pm 0.1) \times 10^3$ kbar and $C_{1111} = +(110 \pm 10) \times 10^3$ kbar. The technique and method of analysis seem generally applicable to solids that exhibit elastic limits of a few percent of their longitudinal elastic constants.

INTRODUCTION

When subjected to shock-wave compression, a number of solids are observed to exhibit unusually large elastic limits. Noting that large elastic compressions can be achieved under shock compression, Fowles[1] expressed the finite-strain high-order elastic constant theory in terms suitable for analysis of elastic shock-compression data. He proposed that longitudinal fourth-order elastic constants could be computed from shock-compression data if the second- and third-order elastic constants were known. From this analysis, the longitudinal fourth-order elastic constants of α quartz were computed from the shock-compression data in the elastic range, i.e., below the Hugoniot elastic limit. The present paper extends the analysis of shock-compression data to the determination of both third- and fourth-order longitudinal elastic constants.

A number of measurements of third-order elastic constants have been accomplished with static compression techniques, including measurements on Ge,[2–6] MgO,[2] Si,[4,6] fused quartz,[2] α quartz,[7] and sapphire.[8] Measurements of these third-order constants are of both fundamental and applied interest. The third-order constants are associated with anharmonicity of a crystal lattice; hence, they may be used to calculate generalized Grüneisen parameters.[9] Furthermore, quantitative descriptions of acoustic amplification at microwave frequencies in solids[10,11] requires knowledge of the third-order elastic constants. If piezoelectric solids are used for amplification, high-order piezoelectric constants are also important.[12–14] The attenuation in microwave delay lines is influenced by the Akhiezer phonon–phonon interaction mechanism,[15] which can be calculated from the third-order elastic constants.

Only a limited number of fourth-order elastic constant measurements have been accomplished and there is no established technique for their determination. In addition to Fowles's measurements, fourth-order constants of several cesium halides have been measured by ultrasonic techniques,[16] and several combinations of fourth-order constants of fused quartz have been determined in uniaxial tension experiments.[17] Fourth-order constants have not yet been required for interpretation of microwave phenomena, but it has been suggested that harmonic generation in stressed crystals could be used to determine fourth-order constants.[18]

Although, at present, only longitudinal elastic constants can be determined from shock-compression measurements, it appears that these longitudinal constants are often of interest. Furthermore, the determination of the third-order constant under large compressions permits a test of the formulation of the finite-strain theory.

Synthetic single-crystal Al_2O_3, i.e., sapphire, exhibits a Hugoniot elastic limit as high as 210 kbar. Hence, high-order longitudinal elastic constants for sapphire can be determined from the shock-compression data. Although third-order constants for sapphire were calculated from shock-compression data in a recent paper,[19] it appears that a more extensive analysis of the data will permit calculation of both third- and fourth-order constants. Furthermore, recent data on fused quartz reported by Barker and Hollenbach[20] permit similar calculations for fused quartz to 38 kbar.

It is the object of this paper to develop a general method for determining third- and fourth-order longitudinal elastic constants from elastic shock-compression data and to use this method to determine the values of these constants for sapphire and fused quartz from the shock-compression data of Graham and Brooks[19] and Barker and Hollenbach.[20] Although the theory expressed by Fowles[1] is used without modification, the method of data analysis is modified so that both third- and fourth-order constants can be determined. The method seems generally applicable to solids that exhibit elastic limits of a few percent of their longitudinal elastic constants.

The paper is organized in the following way: Following the expression of the shock-compression relations and comments on experimental capability, the stress–volume relations are expressed in terms of high-order elastic constants; the method of relating the experimental observations to the high-order constants are then discussed and applied to the shock-compression data; the constants are then compared to similar values obtained ultrasonically from static compression experiments; in conclusion, the general applicability of the method to other solids is discussed.

I. BACKGROUND

A. Shock-Compression Experiments

The most common technique for determining third-order constants involves measurements of the change in transit time of an ultrasonic wave when a solid is subjected to hydrostatic pressures up to about 10 kbar or uniaxial stresses up to about 2 kbar.[8,21] Second-harmonic generation at microwave frequencies has also been used to determine third-order longitudinal constants.[11] Although shock-compression measurements are most frequently accomplished at pressures greater than 100 kbar,[22] techniques utilizing the planar impact of samples with flat-faced projectiles permit shock-compression measurements at stresses as low as 1 kbar.[23,24] When the impact experiment is employed, stresses can be applied to samples on a continuously increasing amplitude scale up to hundreds of kilobars. Instrumentation has been developed that is capable of determining small compression changes and numerous Hugoniot elastic limits have been measured.[25] These observations reveal that a number of solids undergo large compressions while remaining elastic; in particular, Z-cut α quartz has a Hugoniot elastic limit of 150 kbar and X-cut α quartz has a value of 60 kbar, while Z-cut sapphire has a value of 210 kbar and X-cut sapphire has a value of 150 kbar. The possibility of determining high-order constants from shock-compression experiments is a result of the large elastic compressions that some solids exhibit, and the experimental capability for routinely measuring shock compressions throughout the elastic compression range.

When a solid is rapidly compressed ($<10^{-8}$ sec) over a large planar area, the inertial response of the sample produces a well-defined state of one-dimensional strain in the direction of shock propagation.[22] Measurements of shock-wave velocity and the particle velocity behind the shock front are accomplished in a region of the sample that is far enough removed from the boundaries such that unloading from boundaries does not occur in the time of interest to the experiment.

Assuming that materials respond elastically, that the shock profiles are steady in time and one dimensional, and that the shock front moves into an unstressed medium at rest, conservation of momentum[22] results in the equation

$$\sigma_x=\rho_0 U u, \tag{1}$$

where σ_x is the component of stress in the shock propagation direction taken along the x axis, ρ_0 is the initial density, U is the shock-wave velocity, and u is the particle velocity imparted by the shock front. In the low stress limit, $U=(C_{xx}/\rho_0)^{\frac{1}{2}}$, where C_{xx} is the adiabatic second-order longitudinal elastic constant for compression along the direction of wave propagation.

From the conservation of mass, it can be shown that[22]

$$\eta=\Delta V/V_0=u/U, \tag{2}$$

where η is the linear compression, ΔV is the change in specific volume imparted by the shock front, and V_0 is the original specific volume. From these relations, it is apparent that measurements of shock velocity for various particle velocities over a wide range of compressions, up to the Hugoniot elastic limit, permit determination of the elastic-longitudinal-stress versus compression relation. In the elastic range, the stress configuration is well known and the compressions are isentropic to a close approximation.

Various experimental configurations can be utilized to determine the σ_x, η relation. When a projectile impact experiment is utilized, the most effective experimental procedure is to achieve the symmetric impact of the sample with a projectile facing of the same material. Under these conditions, the particle velocity impacted to the sample is exactly one-half the measured projectile impact velocity. The projectile impact velocity can be routinely measured to $\pm 0.1\%$.[26] If explosive loading techniques are utilized, measurements of free surface velocities give a measure of particle velocity to about $\approx 3\%$.